Dick Schaff Math Superbowl XLV
Level 5 Blitz: Secondary Math III – 2018

Directions: (1) Select the most correct answer for each question and mark it on your answer form.
(2) No calculators of any sort are allowed.
(3) Note that N.O.T. means "None of these."

1. Which of the following most closely approximates the number of degrees in half of a radian?

   a) $60°$ b) $90°$ c) $30°$ d) $10°$ e) N.O.T.

2. Let $f(x) = 3x + 7$ and $g(x) = x^2 - 4x + 10$. Determine the value of $f(a) - g(-1)$.

   a) $a^2 - a + 17$ b) $3a - 8$ c) $a^2 - 4a + 14$ d) $3a + 22$ e) N.O.T.

3. Emily has a bag of 40 coins. Only dimes and quarters are found in this bag, and the total value of the coins is \$5.80. Suppose d and q represent the number of dimes and quarters, respectively, in Emily's coin bag. Which of the following systems can be used to find the number of dimes and quarters in Emily's bag?

   a) $\begin{cases} d + q = 5.80 \\ 40d + 40q = 5.80 \end{cases}$ b) $\begin{cases} d + q = 40 \\ 0.25d + 0.10q = 5.80 \end{cases}$

   c) $\begin{cases} d + q = 5.80 \\ 0.10d + 0.25q = 40 \end{cases}$ d) $\begin{cases} d + q = 40 \\ 0.10d + 0.25q = 5.80 \end{cases}$ e) N.O.T.

4. Which of the following is false about the function $f(x) = \log(x)$?

   a) The domain of the function is all real numbers except $x = 0$.
   b) The graph of the function has an x-intercept at the point $(1, 0)$.
   c) The graph of the function has a vertical asymptote at $x = 0$.
   d) The function is always increasing.
   e) N.O.T.

5. For a history test, the mean score was 65, the median score was 71, and the standard deviation of the scores was 7. The teacher decided to add 5 points to each score due to a grading error. Which of the following statements is true?

   a) The new mean score is 70, the new median score is 76, and the new standard deviation is 7.
   b) The new mean score is 70, the new median score is 76, and the new standard deviation is 12.
   c) The new mean score is 65, the new median score is 76, and the new standard deviation is 12.
   d) The new mean score is 65, the new median score is 71, and the new standard deviation is 7.
   e) N.O.T.

6. If $f(x) = \sqrt[3]{x^3+1}$, what is the value of $f^{-1}(4)$?

a) $\sqrt[3]{63}$ b) $\sqrt[3]{65}$ c) $\sqrt[3]{73}$ d) 5 e) N.O.T.

7. If $\log_c(a) = x$, then which of the following is true?

a) $a^c = x$ b) $x^c = a$ c) $a^x = c$ d) $c^a = x$ e) N.O.T.

8. Which of the following trigonometric expressions is not equal to 0?

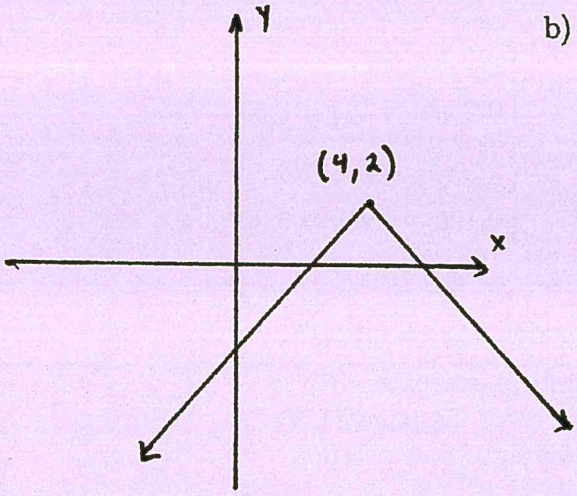
a) $\sin(0)$ b) $\cos\left(\frac{3\pi}{2}\right)$ c) $\tan(\pi)$ d) $\sin(-5\pi)$ e) N.O.T.

9. Which of the following is equivalent to the equation $5\log_8(x) - 2\log_8(y) = 13$?

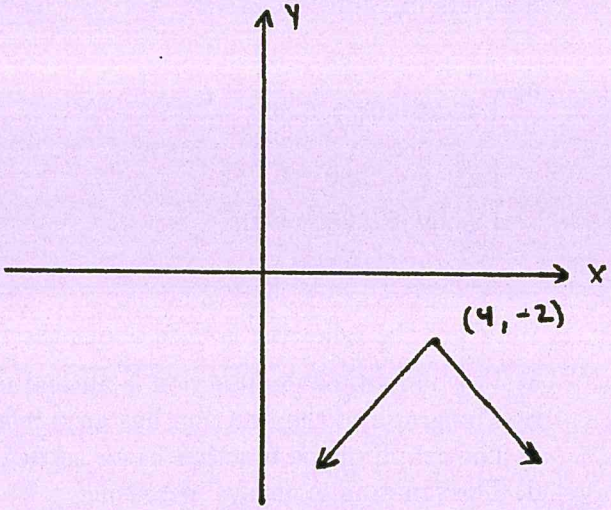
a) $5x - 2y = 8^{13}$ b) $x^5 - y^2 = 8^{13}$ c) $\frac{x^5}{y^2} = 8^{13}$ d) $\frac{5x}{2y} = 8^{13}$ e) N.O.T.

10. Which of the following could be the graph of $y = -|x-4|+2$?

a)



b)



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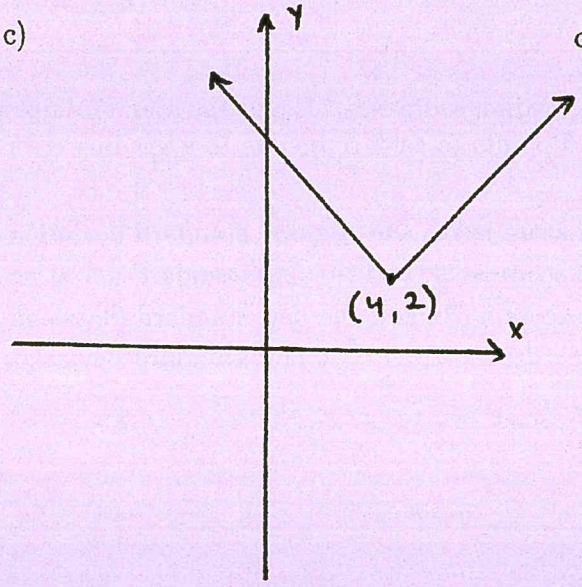

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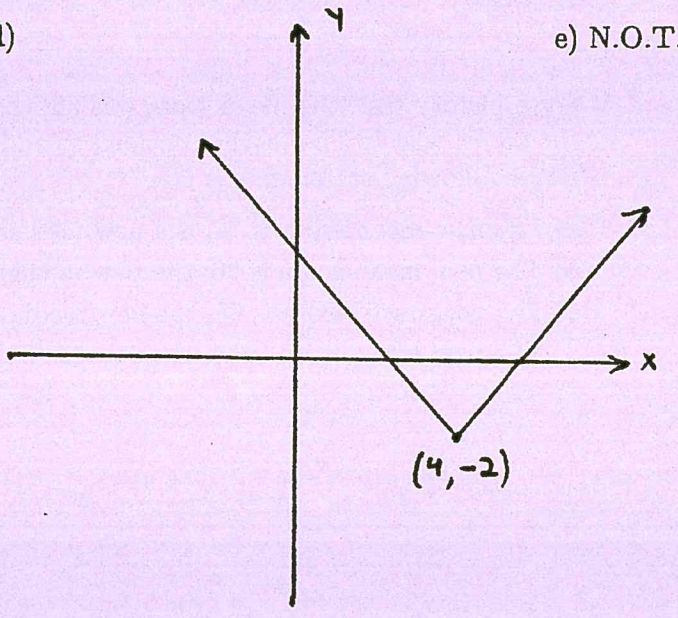
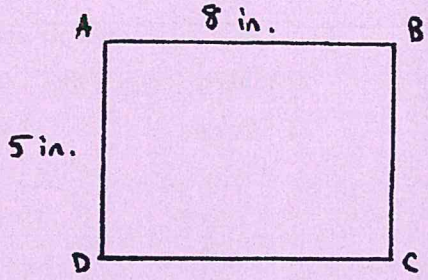


e) N.O.T.

11. The rectangle $ABCD$ shown in the figure is rotated about the side AB. What is the volume of the resulting three-dimensional solid?

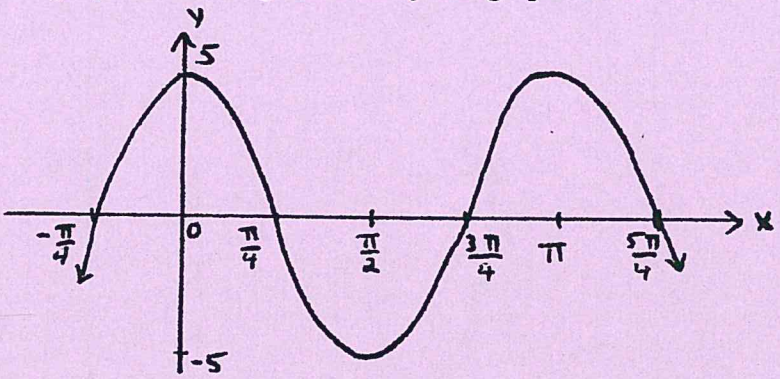


a) 400π in^3 b) 200π in^3 c) 128π in^3
d) 40π in^3 e) N.O.T.

12. For a quadratic equation of the form $ax^2 + by^2 + cx + dy + f = 0$ to represent a parabola, which of the following must happen?

a) Both a and b must be 1.
b) Both c and d must be zero.
c) Either c or d must be zero, but not both.
d) Either a or b must be zero, but not both.
e) N.O.T.

13. Which of the following equations is represented by the graph shown below?



a) $y = 2\sin(5x)$ b) $y = 5\cos(2x)$ c) $y = 2\cos(5x)$ d) $y = 5\sin(2x)$ e) N.O.T.

14. Which of the following polynomials has root $2i$, $-2i$, 1, and -1?

a) $x^4 + 2x^3 + 3x^2 + 8x - 4$ b) $x^4 - 2x^3 + 3x^2 - 8x - 4$
c) $x^4 + 3x^2 - 4$ d) $x^4 - 3x^2 + 4$ e) N.O.T.

15. Which of the following is not true about the function $h(x) = x + 7$?

a) The graph of $h(x)$ can be obtained by shifting the graph of $f(x) = x$ up by 7 units.
b) The graph of $h(x)$ can be obtained by shifting the graph of $f(x) = x$ to the left by 7 units.
c) The graph of $h(x)$ can be obtained by shifting the graph of $f(x) = x$ to the right by 2 units and up by 9 units.
d) The graph of $h(x)$ can be obtained by shifting the graph of $f(x) = x$ to the left by 4 units and up by 3 units.
e) N.O.T.

16. In a circle, a central angle of $\frac{3\pi}{4}$ radians is subtended by an arc of length 18π cm. What is the radius of the circle?

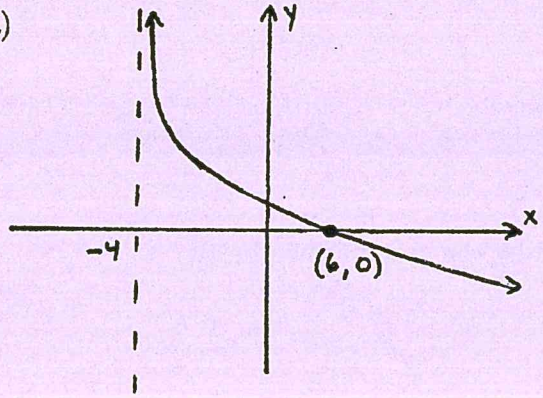
a) 54π cm b) 6π cm c) 13.5 cm d) 24 cm e) N.O.T.

17. If $x-2$ is a factor of $x^3+kx^2+12x-8$, then what must be the value of k?

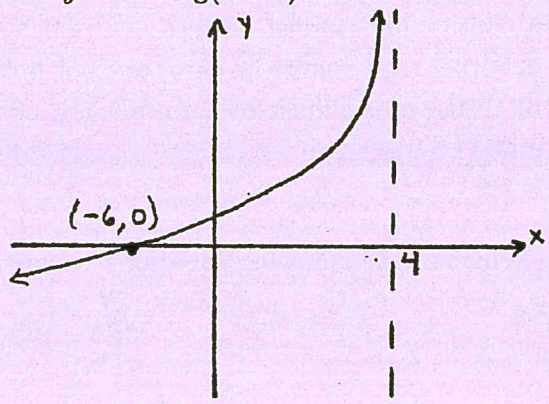
a) -6 b) -3 c) 2 d) 3 e) N.O.T.

18. Which of the following could represent the graph of $y=1-\log(x+4)$?

a)



b)



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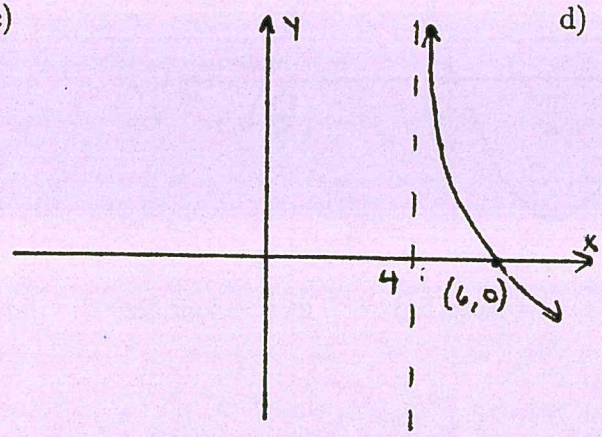
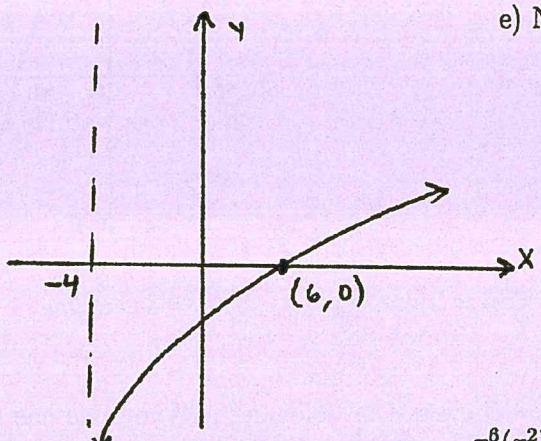


d)



e) N.O.T.

19. Simplify the following expression as far as possible, leaving only positive exponents: $\dfrac{x^6(x^2)^{-4}}{x^{-31}}\cdot x^9$

a) x^{44} b) x^{38} c) $\frac{1}{x^8}$ d) x^{26} e) N.O.T.

20. Suppose $f(x)=x^2+5$ and $g(x)=\sqrt{x+3}$. Which of the following expressions is $g(g(f(x)))$?

a) $\sqrt{x+3}+8$ b) $\sqrt{x^4+16x^2+67}$ c) $\sqrt{\sqrt{x^2+8}+3}$ d) $\sqrt{x+\sqrt{8}+3}$ e) N.O.T.

21. Convert $\frac{\pi}{4}$ radians to degrees.

a) 180° b) 90° c) 60° d) 45° e) N.O.T.

22. A right circular cylinder has radius 3 cm and height 3 cm. If A and B are two points on the surface of the cylinder, what is the maximum possible straight-line distance between A and B?

a) $3\sqrt{2}$ cm b) $3\sqrt{3}$ cm c) 6 cm d) $3\sqrt{5}$ cm e) N.O.T.

23. Suppose the graph of $f(x) = -x^2$ is translated 3 units to the left and 1 unit up. If the resulting graph represents $g(x)$, what is the value of $g(4)$?

a) 0 b) -2 c) 50 d) -13 e) N.O.T.

24. Which of the following is not an even function?

a) $f(x) = x^4$ b) $g(x) = x^4 - 5$ c) $h(x) = (x-5)^4$ d) $k(x) = -5x^4$ e) N.O.T.

25. Determine all solutions of $2\log_5(x) = 3\log_5(4)$.

a) $x = \pm 8$ b) $x = 6$ c) $x = 8$ d) $x = 32$ e) N.O.T.

26. Which of the following is the result of dividing $2x^2 - 1$ by $x + 2$?

a) $2x - 8 + \dfrac{17}{x+2}$ b) $2x - 4 + \dfrac{7}{x+2}$

c) $-2x + 4 - \dfrac{9}{x+2}$ d) $-2x + 8 - \dfrac{5}{x+2}$ e) N.O.T.

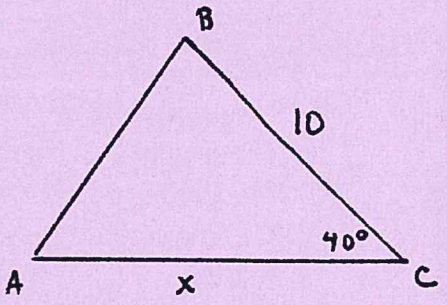
27. Find the coefficient of the 5th degree term in the binomial expansion of $(x-2)^7$.

a) 84 b) 21 c) -14 d) -672 e) N.O.T.

28. In the triangle to the right, the lengths of AB and BC are the same. Which of the following represents the exact value of x?

a) $\dfrac{10\sin(40°)}{\sin(100°)}$ b) $\dfrac{\sin(80°)}{10\sin(40°)}$ c) $\dfrac{10\sin(100°)}{\sin(40°)}$

d) $\dfrac{\sin(100°)}{10\sin(80°)}$ e) N.O.T.



29. A sequence is defined recursively by $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. If $a_1 = 0$ and $a_2 = 1$, what is the value of a_6?

a) 5 b) 8 c) 11 d) 13 e) N.O.T.

30. In a math class at a local college, the class average was 70 out of 100 points, with a standard deviation of 8 points. The scores were distributed normally. What percentage of students earned a score between 70 and 86?

a) 47.5% b) 34% c) 95% d) 68% e) N.O.T.

31. What is the diameter of a circle that has an area of 81π cm^2?

a) 9 cm b) 4.5 cm c) 12 cm d) 18 cm e) N.O.T.

32. Determine the solution set of $4 - x^2 \geq x - 2$.

a) $(-\infty, -2] \cup [3, \infty)$ b) $(-2, 3)$
c) $[-3, 2]$ d) $(-\infty, -3) \cup (2, \infty)$ e) N.O.T.

33. If $f(x) = x - x^2$, then $\dfrac{f(x+h) - f(x)}{h}$ is:

a) 1 b) $1 + 2x - h$
c) $\dfrac{2x + h + 2x^2 - 2xh + h^2}{h}$ d) $\dfrac{2x^2 + 2h^2}{h}$ e) N.O.T.

34. Which of the following is a root of the function $f(x) = \dfrac{x^2 + 4x - 21}{x - 3}$?

a) -3 b) 3 c) 7 d) -7 e) N.O.T.

35. Which of the following geometric shapes cannot be a cross-section obtained by the intersection of a plane and a right circular cylinder with finite height?

a) Circle b) Parabola c) Ellipse d) Rectangle e) N.O.T.

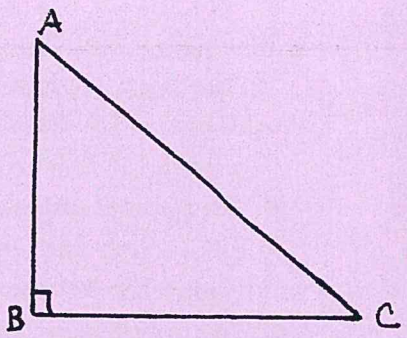
36. Let $f(x) = x + 3$ and $g(x) = \dfrac{x^2 - 9}{x - 3}$. Which of the following is true about the graphs of f and g?

a) The graphs are exactly the same.
b) The graphs are exactly the same except at $x = -3$.
c) The graphs are exactly the same except at $x = -3$ and $x = 3$.
d) The graphs have no points in common.
e) N.O.T.

37. Which of the following must be true about a function $f(x)$ and its inverse $f^{-1}(x)$?

a) If f is an increasing function, then f^{-1} must be a decreasing function.
b) If f has an x-intercept at $(3, 0)$, then f^{-1} has a y-intercept at $(0, -3)$.
c) If f has domain $[0, 5]$, then f^{-1} has range $[-5, 0]$.
d) The graphs of f and f^{-1} can never intersect.
e) N.O.T.

38. Suppose the right triangle shown in the figure is revolved about the side $\overline{AB}$. What three-dimensional shape would be formed?



a) Cylinder
b) Square pyramid
c) Cone
d) Square frustrum
e) N.O.T.

39. In a normally distributed data set, which of the following accurately describes the meaning of a z-score of -1.2 for a given observation?

a) The observation is 1.2 standard deviations above the mean of the data set.
b) The data set has a mean of -1.2.
c) The data set has a standard deviation of 1.2.
d) The smallest data value in the data set is -1.2.
e) N.O.T.

40. The line with equation $y = 7$ is graphed on the same xy-plane as the circle with center $(4, 5)$ and radius 3. What are the x-coordinates of the points of intersection of the line and the circle?

a) $x = 4 \pm \sqrt{7}$
b) $x = 1,\ 7$
c) $x = 2,\ 8$
d) $x = 4 \pm \sqrt{5}$
e) N.O.T.

41. Simplify as far as possible: $\dfrac{x^2+10x+24}{x^2-3x+2} \times \dfrac{x^2+x-6}{x^2+9x+18}$.

a) $\dfrac{x+4}{x-1}$
b) $\dfrac{(x+4)(x+6)}{(x+1)(x-6)}$
c) $\dfrac{x-4}{x+1}$
d) $\dfrac{(x+3)(x+6)}{(x-6)(x-2)}$
e) N.O.T.

42. The domain of the function $g(x) = \sqrt{\dfrac{x-7}{x+4}}$ is:

a) $(-\infty, -4) \cup [7, \infty)$
b) $(-4, 7]$
c) $(-\infty, -4] \cup (7, \infty)$
d) $(-\infty, -4) \cup (-4, \infty)$
e) N.O.T.

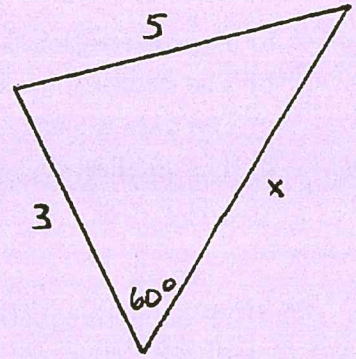
43. A politician is interested in the opinions of the people in her district, so she selects five different neighborhoods and interviews everyone in them. What type of sample did this politician take?

a) Simple random sample
b) Voluntary response
c) Cluster sample
d) Systemic sample
e) N.O.T.

44. The same politician as in #43 is now interested in how a particular city's obesity rate compares to the state average. So, she goes to a local grocery store, randomly selects 100 city residents, and asks them questions regarding their diet, height, weight, and exercise routine. Which of the following statements about this study is false?

a) The population is all of the people in the city.
b) The sample was everyone at the grocery store.
c) The parameter was the obesity rate of the city.
d) The information was collected by a survey.
e) N.O.T.

45. Using the information in the triangle shown in the figure to the right, determine the side length x.



a) $\frac{3+\sqrt{73}}{2}$ b) $\frac{16+\sqrt{21}}{2}$

c) $\frac{1+\sqrt{73}}{32}$ d) 4 e) N.O.T.